



WINTHROP FORENSICS

Quarterly

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Winthrop Forensics
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Accident Reconstruction
Biomechanical / Injury Causation Analysis
Premises Liability, Product Defects
Forensic Exhibits and Animations



In this installment, *The Statistical Edge* presents some interesting American Sports Related Eye Injury statistical data. In the *Physics In ACTION* article, we present the second of a three part series that describes Projectile Motion. This second installment will deal with the case of air resistance (drag) imparted to the projectile. Lastly, some Halloween Trivia is provided for your enjoyment.



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HALLOWEEN TRIVIA

Halloween is near! With this in mind, we deliver you a little bit of Halloween Trivia.

- Which region in the world do Pumpkins originate from?
A) South America B) Central America C) France
- Who wrote the novel "Frankenstein"?
A) Mary Shelley B) Mark Twain
C) Orson Wells
- Is a pumpkin a fruit or vegetable?
- Which country celebrates Day of the dead on Nov. 1st?
A) Venezuela B) Germany C) Mexico
- ORANGE** and **BLACK** are Halloween colors because:
A) Orange is associated with the Fall harvest and black is associated with darkness and death.
B) Orange is associated with Jack-O-Lanterns and black is associated with the Black Cat.
C) Orange is associated with Good Spirits and black is associated with Evil Spirits.
- Halloween is the _____ most commercially successful holiday.
A) First B) Second C) Third D) Fourth
- Chocolate candy bars top the list as the most popular candy for trick-or-treaters with:
A) Hershey's Bar B) Twix Bar C) Snickers Bar
- How many Americans decorate their house for Halloween?
A) 24% B) 55% C) 86% D) 92%
- Over ____ of pet owners dress their pets in a Halloween costume.
A) 5% B) 10% C) 15% D) 20%
- October 30th is National Candy Corn Day. Americans consume _____ pounds of candy corn each year:
A) 5 million B) 10 million
C) 20 million D) Over 30 million



Physics in ACTION

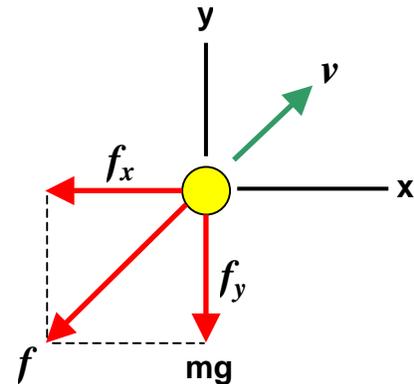
Projectile Motion - Part 2 of 3

This article is the second of the three part series that describes Projectile Motion. This installment will deal with the case of air resistance (drag) imparted to the projectile.

For this case, a projectile is defined as an object that moves in two dimensions (planar motion) with no means of propulsion. The object's path is defined as the trajectory of the projectile.

Example: Two Dimensional Soft Ball Flight

As with the no drag case, the two dimensional projectile trajectory can be fully analyzed by breaking the planar motion up into two cases of one dimensional motion. The first case is motion along the x-axis, the second case is motion along the y-axis.



DERIVATIONS

At the velocity v of the projectile, the magnitude of the drag force D is approximately proportional to the square of the projectile's velocity relative to the air:

$$f = Dv^2 \quad (1)$$

where

$$v = v_x^2 + v_y^2 \quad (2)$$

The components of f are:

$$f_x = -Dvv_x \quad \text{and} \quad f_y = -Dvv_y \quad (3)$$

Application of Newton's 2nd law ($F = ma$) to the above free body diagram and Equation 3 yields ($m = \text{mass}$):

$$\sum F_x = -Dvv_x = ma_x \quad (4)$$

$$\sum F_y = -mg - Dvv_y = ma_y \quad (5)$$

Solving for the x-direction and y-direction components of acceleration in Equations 4 and 5, which include the effects of both gravity and air drag yields:

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$$a_x = -(D/m) v v_x \quad (6)$$

$$a_y = -(D/m) v v_y - g \quad (7)$$

The constant **D** is a function of the density of air ρ , the silhouette area **A** of the projectile (its area as seen from the front), and a dimensionless constant **C** called the drag coefficient. The drag coefficient **C** depends on the shape of the projectile.

Typical values of **C** for baseballs, softballs, tennis balls, and the like are in the range from 0.2 to 1.0. In terms of these quantities, **D** is given by:

$$D = \frac{\rho CA}{2} \quad (8)$$

At this point, we have the required relationships in terms of physical equations. In order to solve for the projectile motion, we use these relationships for the numerical calculation.

NUMERICAL CALCULATION EQUATIONS

During a time interval Δt , the average x-component of acceleration is $a_x = \Delta v_x / \Delta t$ and the x-velocity v_x changes by an amount $\Delta v_x = a_x \Delta t$. Similarly, the average y-component of acceleration is $a_y = \Delta v_y / \Delta t$ and the y-velocity v_y changes by an amount $\Delta v_y = a_y \Delta t$. The values of the x-velocity and y-velocity at the end of the interval are:

$$v_x + \Delta v_x = v_x + a_x \Delta t \quad (9)$$

$$v_y + \Delta v_y = v_y + a_y \Delta t \quad (10)$$

The average x-velocity during the time interval Δt is the average of the value v_x (at the beginning of the interval) and $v_x + \Delta v_x$ (at the end of the interval), or $v_x + \Delta v_x / 2$. Using Equation 9, during Δt the coordinate x changes by an amount:

$$\Delta_x = (v_x + \Delta v_x / 2) \Delta t = v_x \Delta t + \frac{1}{2} a_x (\Delta t)^2 \quad (11)$$

And using Equation 10, the coordinate y changes by:

$$\Delta_y = (v_y + \Delta v_y / 2) \Delta t = v_y \Delta t + \frac{1}{2} a_y (\Delta t)^2 \quad (12)$$

NUMERICAL CALCULATIONS

All of the relationships required for the numerical calculation are present. A convenient tool for the calculation is MS Excel® as outlined in the following steps. For this example, we will use a softball with an initial velocity of $v_o = 40$ mph thrown at an angle of $\theta = 45^\circ$.

Step 1 Identify the initial parameters m, A, r, C, then calculate D.

For the softball: $A = \pi r^2$; r = radius = 0.145 ft
 $A = 0.066 \text{ ft}^2$
 $C = 0.5$
 $m = 0.01165 \text{ slug}$
 $\rho = 0.002378 \text{ slug/ft}^3$
 $D = 3.93E-5$

Step 2 At time t = 0, calculate the initial values of x and y, v_x and v_y .
 For this example, $x(0) = y(0) = 0$ and:

$$v_x(0) = v_o \cos(\theta) = 41.5 \text{ ft/s} \text{ and}$$

$$v_y(0) = v_o \sin(\theta) = 41.5 \text{ ft/s}$$

Step 3 Choose a Δt and iterate stepwise to desired end $x(n)$ and $y(n)$ values. For this case, we will choose a value of $\Delta t = 0.1$.

For the first iteration, we calculate $a_x(n)$ and $a_y(n)$, $v_x(n)$ and $v_y(n)$, $x(n)$ and $y(n)$ where $n=1$.

From Equation 6 and 7:

$$a_x(n) = -(D/m) v^{(n-1)} v_x(n-1) = -8.2 \text{ ft/s}^2$$

$$a_y(n) = -(D/m) v^{(n-1)} v_y(n-1) = -40.4 \text{ ft/s}^2$$

From Equations 9 and 10:

$$v_x(n) = v_x(n-1) + a_x(n) \Delta t = 40.68 \text{ ft/s}$$

$$v_y(n) = v_y(n-1) + a_y(n) \Delta t = 37.46 \text{ ft/s}$$

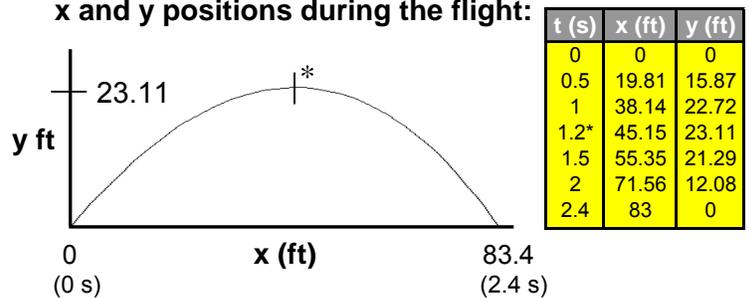
From Equations 11 and 12:

$$x(n) = x(n-1) + v_x(n) \Delta t + (1/2) a_x(n) (\Delta t)^2 = 4.1 \text{ ft}$$

$$y(n) = y(n-1) + v_y(n) \Delta t + (1/2) a_y(n) (\Delta t)^2 = 3.95 \text{ ft}$$

Step 4 For the subsequent iterations, we increment n and recalculate $a_x(n)$ and $a_y(n)$, $v_x(n)$ and $v_y(n)$, and $x(n)$ and $y(n)$ where, then increment n.

x and y positions during the flight:



APPLICATIONS

This two dimensional trajectory model is an ideal analysis tool for many cases where air resistance can be assumed to be a function of the projectile's front profile.

DRAG versus NO DRAG COMPARISONS

This is the topic of **Part 3** - the third installment in this series which will be presented in the next issue of *WFQ*.

The Statistical Edge

American Sports Related Eye Injuries

It is estimated that 1.6 to 2.4 million people sustain eye injury each year. Sports related eye injuries in the United States account for more than 100,000 physician visits yearly at a cost of \$175 million. More than 42,000 of these sports related eye injuries require a visit to an emergency room.



Every **13 minutes** an emergency room in the United States treats sports related eye injury.

More than **100,000** eye injuries are estimated to be sports related.

This represents greater than **6.3%** of the total eye injury population.

BLINDNESS

- Approximately **33%** of eye injuries leading to blindness are sports related.
- It is estimated that **1.7% to 2.5%** of people who sustain eye injuries each year will be legally blinded in the injured eye.

CHILDREN

- Children under the age of 15 account for **43%** percent of sports and recreational eye injuries overall.
- An estimated **27%** percent of all eye injuries in children aged 11 to 14 are sports-related.



PROTECTIVE EYEWEAR



- In 2002, **15%** of children and **33%** of adults reported wearing eye protection always or most of the time when participating in sports, hobbies, or other activities that can cause eye injuries.

- **90%** of eye injuries of school aged children can be avoided with protective eye wear.

These injuries are the leading cause of blindness in this age group.

BASKETBALL

- Basketball is a leading cause of sports related eye injury in athletes aged **15 to 24**.
- The odds of an eye injury for basketball players are 1 in 10.15 (**9.5%** of players).
- Of 1,092 injuries sustained by National Basketball Association players during a 17 month period in 1992 and 1993, **5.4%** involved the eye, and **15.3%** missed subsequent games because of their injury.



96.6% of the injured players were not wearing protective eyewear at the time of injury.

GOLF



- Golf related ocular injuries account for **1.5% to 5.6%** of all sports injuries.

